VMC Vidyamandir Classes SINCE 1986
IIT JEE | MEDICAL | FOUNDATION

CODE A

JEE Advanced Home Practice Test -4 | Paper -1 | JEE 2024

Date: 30/04/2024 **Maximum Marks: 180**

Duration : 3.0 Hours

General Instructions

1. The question paper consists of 3 Subject (Subject I: **Physics**, Subject II: **Chemistry**, Subject III: **Mathematics**). Each Part has **Four** sections (Section 1, Section 2, Section 3 and Section 4).

2. **Section 1** contains **4 Single Correct Answer Type Questions**. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.

3. **Section 2** contains **3 Question stems**. There are **TWO (02)** questions corresponding to each question stem. For each question, enter the correct numerical value of the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/roundoff** the value to **TWO** decimal places.

4. **Section 3** contains **6 Multiple Correct Answer Type Questions.** Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).

5. **Section 4** contains **3 Non-Negative Integer Type Questions.** The answer to each question is a **NON-NEGATIVE INTEGER.**

6. For answering a question, an ANSWER SHEET (OMR SHEET) is provided separately. Please fill your **Test Code, Roll No.** and **Group** properly in the space given in the ANSWER SHEET.

Name of the Candidate (In CAPITALS) : _____

Roll Number : _____

OMR Bar Code Number : _____

Candidate's Signature : _____ **Invigilator's Signature** _____

MARKING SCHEME

SECTION – 1 | (Maximum Marks: 12)

- This section contains **Four (04) Single Correct Answer Type Questions**. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme.

Full Marks : +3 If ONLY the correct option is chosen.

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).

Negative Marks : −1 In all other cases.

SECTION – 2 | (Maximum Marks: 12)

- This **Section** contains **3 Question stems**. There are **TWO (02)** questions corresponding to each question stem. For each question, enter the correct numerical value of the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/roundoff** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks: +2 If ONLY the correct numerical value is entered at the designated place.

Zero Mark: 0 In all other cases.

SECTION – 3 | (Maximum Marks: 24)

- This section consists of **Six (06)** Questions. Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks: +4 If only (all) the correct option(s) is(are) chosen

Partial Marks: +3 If all the four options are correct but ONLY three options are chosen

Partial Marks: +2 If three or more options are correct but ONLY two options are chosen and both of which are correct

Partial Marks: +1 If two or more options are correct but ONLY one option is chosen, and it is a correct option

Zero Mark: 0 if none of the options is chosen (i.e. the question is unanswered)

Negative Marks: –2 In all other cases.

SECTION – 4 | (Maximum Marks: 12)

- This **Section** contains **3 Non-Negative Integer Type Questions.** The answer to each question is a **NON-NEGATIVE INTEGER**
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme.

Full Marks : +4 If ONLY the correct integer is entered.

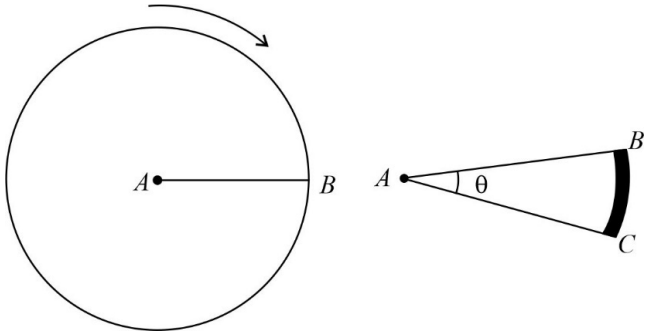
Zero Marks : 0 In all other cases.

SUBJECT I : PHYSICS 60 MARKS

Section – 1 | Single Correct Type

This Section contains **4 Single Correct Answer Type Questions**. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.

1. Two-point sources of light are fixed at the centre (A) and circumference $(point\ B)$ of a rotating turn table. A photograph of the rotating table is taken. On the photograph a point A and an arc BC appear. The angle θ was measured to be $\theta = 10.8° \pm 0.1°$ and the angular speed of the turntable was measured to be $\omega = (33.3 \pm 0.1)$ revolution per minute. Calculate the exposure time of the camera.



(A) $(0.054 \pm 0.003)s$ **(B)** $(0.054 \pm 0.004)s$

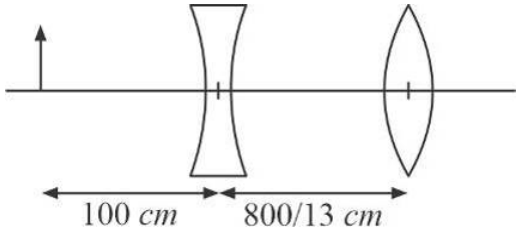
(C) $(0.054 \pm 0.005)s$ **(D)** $(0.054 \pm 0.002)s$

2. At a depth $h_1 = \frac{R}{2}$ from the surface of the earth, acceleration due to gravity is g_1. Its value changes by Δg_1 when one moves down further by 1 km. At a height h_2 above the surface of the earth, acceleration due to gravity is g_2. Its value changes by Δg_2 when one moves up further by 1 *km*. If $\Delta g_1 = \Delta g_2$, find h_2. Assume the earth to be a uniform sphere of radius R.

(A) $h_2 = R[2^{2/3} - 1]$ **(B)** $h_2 = 3R[2^{1/3} - 1]$

(C) $h_2 = 2R[2^{1/3} - 1]$ **(D)** $h_2 = R[2^{1/3} - 1]$

3. A concave lens and convex lens are placed as shown in figure. The refractive index of concave and convex lens is 7/5 and 3/2 respectively and radii of all curved surfaces are R = 50 *cm*. An object is placed at 100 *cm* from concave lens then final location of image is:



(A) At infinity

(B) At 100 *cm* from convex lens

(C) At 200 *cm* from concave lens

(D) At 750/3 *cm* from convex lens

4. Nuclei of radioactive element A are produced at a rate 't^2' at any time t. The element A has decay constant λ. Let N be the number of nuclei of element A at any time t. At time $t = t_0$, dN/dt is minimum. Then the number of nuclei of element A at time $t = t_0$ is:

(A) $\frac{t_0 - \lambda t_0^2}{\lambda^2}$ **(B)** $\frac{\lambda t_0^2 - 2t_0}{\lambda^2}$ **(C)** $\frac{2t_0 - \lambda t_0^2}{\lambda}$ **(D)** $\frac{t_0 - \lambda t_0^2}{\lambda}$

Section – 2 | Numerical Value Type

This **Section** contains **3 Question stems**. There are **TWO (02)** questions corresponding to each question stem. For each question, enter the correct numerical value of the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/roundoff** the value to **TWO** decimal places.

Question Stem: For Question number. 5 to 6

Question Stem

A ball of mass 'm' is projected at angle of 30° with velocity $10\,m/\sec$. At the top of journey, it explodes into two pieces of equal mass. When both the fragments landed, their combined kinetic energy was observed to be 50% more than kinetic energy of ball at the time of projection. Both the fragments land at the same moment and after covering same horizontal distance along the vertical plane of projection. At the time of landing, the distance between fragments is $\sqrt{x}$ metre and speed of one fragment is $v = \sqrt{k}$ m/s then: (take $g = 10 m/s^2$)

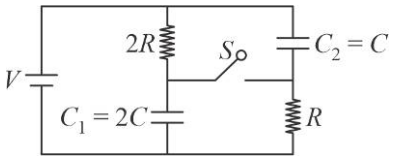
5. The value of x is _____.

6. The value of k is _____.

Question Stem: For Question number. 7 to 8

Question Stem

Initially switch 'S' is closed for long time. Now at t = 0 switch is opened. The emf of battery is $V = 12V$, $R = 4\,\Omega$ and capacitance $C = 1\,\mu F$.



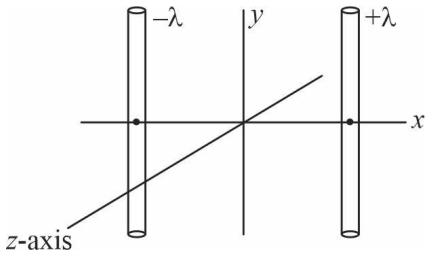
7. The current through 'C_2' at $t = RC\ln(2)$ is _____*Amp.*

8. Charge on 'C_1' just before time t = 0 is _____ μC.

Question Stem: For Question number 9 to 10

Question Stem

Two long wires of charge density '$+\lambda$' and '$-\lambda$' are parallel to y-axis and passing through the points $(a, 0, 0)$ and $(-a, 0, 0)$ respectively . A point charge 'Q' is placed at (0, 0, 0) and displaced slowly.
$(\lambda = 40\mu C/m,\ Q = 1\mu C,\ ln(3) = 1.1)$



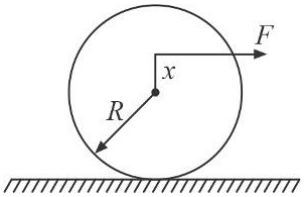
9. Amount of work-done to move the charge Q from (0, 0, 0) to $\left(\frac{a}{2}, 0, 0\right)$ is _____ J.

10. Amount of work-done to move the charge Q from (0, 0, 0) to $\left(0, 0, \frac{a}{2}\right)$ is _____ J.

Section – 3 | Multiple Correct Type

This Section contains **6 Multiple Correct Answer Type Questions.** Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).

11. A horizontal force 'F' is being applied above height 'x' from the center of body. The mass of body is M and radius is 'R'. Consider the ground to be sufficiently rough. Which of the following statement(s) is(are) correct?



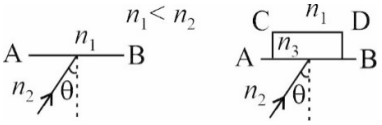
(A) For $x = R$, for solid sphere, it has $a_{COM} = \frac{10}{7}\frac{F}{M}$

(B) For $x = R$, for solid sphere, the friction acts in forward direction and $f = \frac{3}{7}F$

(C) For $x = \frac{R}{2}$, for solid cylinder, it has $a_{COM} = \frac{F}{M}$

(D) For $x = \frac{R}{2}$, for solid cylinder, the friction vanishes

12. In the figure, light is incident at an angle θ which is slightly greater than the critical angle. Now, keeping the incident angle fixed, a parallel slab of refractive index n_3 is placed on surface AB. Which of the following statement(s) is(are) correct?



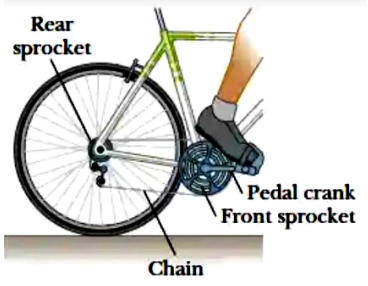
(A) total internal reflection occurs at AB for $n_3 < n_1$

(B) total internal reflection occurs at AB for $n_3 > n_1$

(C) the ray will return back to the same medium for all values of n_3

(D) total internal reflection occurs at CD for $n_3 > n_1$

13. Figure shows the drive train of a bicycle that has wheels 80 cm in diameter. The cyclist pedals at a steady cadence of 75 rev/min. The chain engages with a front sprocket 16 cm in diameter and a rear sprocket 8.0 cm in diameter. Which of the following statements are true?



(A) The speed of a link of the chain relative to the bicycle frame is $\frac{\pi}{5} m/s$

(B) The angular speed of the bicycle wheels is 5π rad/ s

(C) The speed of the bicycle relative to the road is $2\pi m/s$

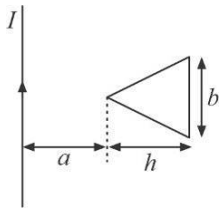
(D) The speed of the bicycle relative to the road is $4\pi m/s$

14. Consider a Bohr orbit in the hydrogen atom with principal quantum number $n(n > 1)$. If an electron in this orbit makes a transition to the immediately lower orbit, it emits a photon of wavelength λ_L and the change in momentum experienced by the hydrogen atom due to this emission be Δp_L. Let the largest wavelength of a photon that can ionize this hydrogen atom (with the electron now in an orbit with principal quantum number $(n-1)$ be λ_M. Choose the correct option(s):

(A) λ_L is proportional to $\dfrac{(n(n-1))^2}{(2n-1)}$ **(B)** Δp_L is proportional to $\dfrac{1}{(n-1)^2}$

(C) Δp_L is proportional to $\dfrac{(2n-1)}{(n(n-1))^2}$ **(D)** λ_M is proportional to $(n^2-1)^2$

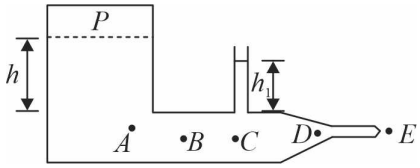
15. A very long straight conductor and an isosceles triangular conductor lie in a plane and are separated from each other as shown in figure. Here $a = 10\ cm$, $b = 20\ cm$ and $h = 10\ cm$. If $\phi =$ flux through isosceles triangular conductor and M be the mutual inductance then choose the correct option(s).



(A) $\phi = \dfrac{\mu_0 i}{2\pi}\dfrac{b}{h}[h - a\ln 2]$ **(B)** $\phi = \dfrac{\mu_0 i}{2\pi}\dfrac{b}{h}[h + a\ln 2]$

(C) $M = 4\times 10^{-8}\,[1+\ln 2]$ **(D)** $M = 4\times 10^{-8}\,[1-\ln 2]$

16. As shown in the figure a liquid of density is ρ standing in a sealed container to a height h. The container contains compressed air at a gauge pressure of p. The horizontal outlet pipe has a cross-sectional area $A/2$ at E and area A at C. Which of the following statement(s) is(are) correct?



(A) The velocity of liquid at C will be $\left[\dfrac{(p+\rho gh)}{4\rho}\right]^{1/2}$

(B) The velocity of liquid at C will be $\left[\dfrac{2(p+\rho gh)}{\rho}\right]^{1/2}$

(C) The discharge rate is given by $\dfrac{A}{2}\left[\dfrac{p+\rho gh}{\rho}\right]^{1/2}$

(D) The discharge rate is given by $\dfrac{A}{2\sqrt{\rho}}\left[p+\rho gh\right]^{1/2}$

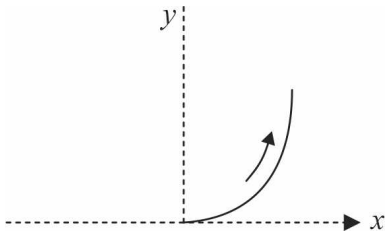
SPACE FOR ROUGH WORK

Section – 4 | Non-Negative Integer Type

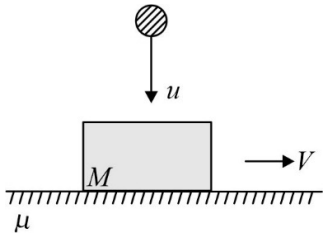
This **Section** contains **3 Non-Negative Integer Type Questions.** The answer to each question is a **NON-NEGATIVE INTEGER.**

17. Suppose a hypothetical magnetic field exists in space $\vec{B} = B_0\hat{u}_r$ above the earth surface where $\hat{u}_r$ is a unit vector directed radially outward from origin. Origin is on surface of earth. A light charged particle has to perform uniform circular motion in the combined uniform (vertical) gravitational field of earth and magnetic field with speed *v* and radius *r*. Height of the plane of motion from earth surface will be $h = \frac{nv^2}{g}$. Find *n*.

18. In order to simulate different values of *g*, aspiring astronauts are put on a plane which dives in a parabola given by the equation $x^2 = 500y$, where *x* is horizontal and *y* is vertically upwards, both being measured in meter. The *x*-component of velocity of the plane is constant and has the value of $360\,km\,h^{-1}$. The effective value of *g* experienced by an astronaut on the plane is βg. Find the value of β.



19. A block of mass $M = 5kg$ is moving on a horizontal table and the coefficient of friction is $\mu = 0.4$. A clay ball of mass $m = 1kg$ is dropped on the block, hitting it with a vertical velocity of $u = 10\ m/s$. At the instant of hit, the block was having a horizontal velocity of $v = 2m/s$. After an interval of Δt, another similar clay ball hits the block and the system comes to rest immediately after the hit. Assume that the clay balls stick to the block and collision is momentary. If $\Delta t = \frac{1}{2n}$ second, find the value of *n*. (Take $g = 10m/s^2$)



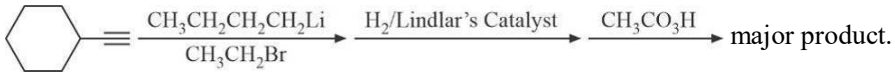
SPACE FOR ROUGH WORK

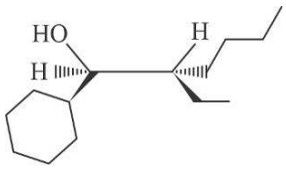
SUBJECT II : CHEMISTRY 60 MARKS

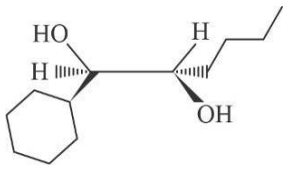
Section – 1 | Single Correct Type

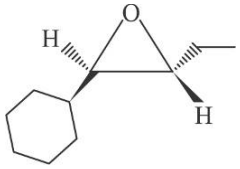
This Section contains **4 Single Correct Answer Type Questions**. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.

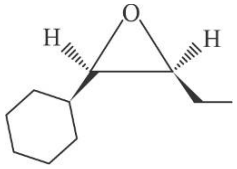
20. Give the major product of the following reaction sequence.



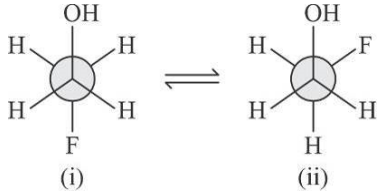
(A)


(B)


(C)


(D)


21. What is true about the following two conformers of 2-fluoro ethanol?



(A) Both have equal percentage at equilibrium

(B) (i) has greater percentage at equilibrium due to less steric strain

(C) (ii) has greater percentage at equilibrium due to intra-molecular H-bonding

(D) Transformation of (i) to (ii) will be an endothermic process

22. Packing fraction of cubic lattice of diamond is _____.

(A) 0.78 **(B)** 0.68 **(C)** 0.34 **(D)** 0.52

23. The calculated spin only magnetic moments of $[Co(H_2O)_6]^{2+}$ and $[Fe(NH_3)_6]^{2+}$ in BM, respectively, are: [Atomic number of Co and Fe are 27 and 26 respectively].

(A) 1.73 and 4.89 **(B)** 2.84 and 3.87

(C) 3.87 and 2.84 **(D)** 3.87 and 4.89

SPACE FOR ROUGH WORK

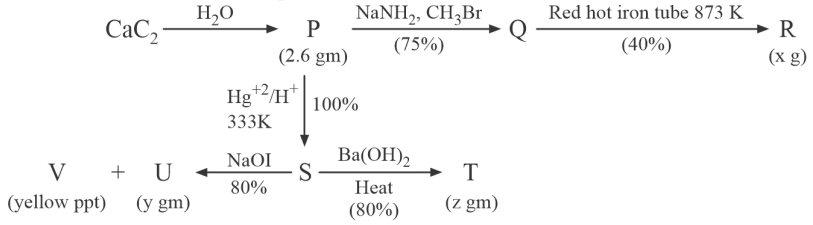
Section – 2 | Numerical Value Type

This **Section** contains **3 Question stems**. There are **TWO (02)** questions corresponding to each question stem. For each question, enter the correct numerical value of the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/roundoff** the value to **TWO** decimal places.

Question Stem: For Question number. 24 to 25

Question Stem

For the following reaction scheme, percentage yields are given along the arrow:



P is an organic compound and empirical formula of R is C_2H_3.

x gm, y gm and z gm are mass of R, U and T respectively.

(Use : Molar mass (in gm mol^{-1}) of H, C and O as 1, 12 and 16 respectively).

24. The value of x is _____.

25. The value of (y + z) is _____.

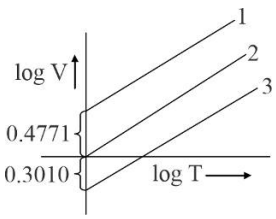
Question Stem: For Question number. 26 to 27

Question Stem

Ideal gases obey the equation $PV = nRT$, where the symbols have their usual meaning. Rate of effusion of ideal gases is proportional to $\frac{P}{\sqrt{M^\circ}}$. Where M° is gram molar mass.

26. At constant P of 0.0821 atm, logV v/s log T graph is plotted for 3 samples of ideal gas as shown. Values of number of moles of gases in these three samples are x_1 (for gas 1), x_2 (for gas 2) and x_3 (for gas 3) respectively, then what is the value of $(x_1 + x_2 + x_3)$?

$(\text{Take } R - 0.0821 atm - L/mole - K, \log_{10} 2 = 0.3010, \log_{10} 3 = 0.4771)$



27. Fractional diffusion method can be used to separate different isotopes of an element. An equimolar mixture of hydrogen and deuterium gases was subjected to diffuse through pin holes in multiple steps to create a 8 : 1 molar ratio of these gases. Find number of steps required to achieve that molar ratio.

Question Stem: For Question Nos. 28 to 29

Question Stem

An isotope of hydrogen atom is represented as X which follows Bohr's model and exists as diatomic gaseous molecule X_2. Also the normal boiling point of a compound X_2O liquid is found to be 101°C and that of a solution obtained on dissolving 0.1 moles of NaCl in 1 kg X_2O liquid is 101.4°C. It is also known that the ionization energy of X is equal to 18 eV. Assume complete dissociation of NaCl in X_2O.

28. The value of ebullioscopic constant (in $K\ kg\ mol^{-1}$) of X_2O is given by_____.

29. The energy (in eV/atm) required to excite electron from ground state to IInd excited state is given by:

Section – 3 | Multiple Correct Type

This Section contains **6 Multiple Correct Answer Type Questions.** Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).

30. Which is/are true regarding D-glucose and D-fructose?

(A) Both forms same osazone with $C_6H_5NHNH_2$

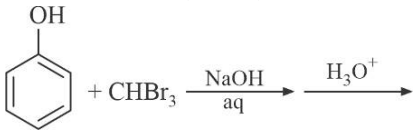
(B) Both should be dextro sugars only.

(C) Both forms a monocarboxylic acid with $Br_2 - H_2O$

(D) Reduction product of D-glucose is one of the epimer obtained on reduction of D-fructose

31. Consider the following reaction,

The correct statement regarding above reaction is/are:



(A) Phenol undergo electrophilic attack by CBr_2

(B) Electrophile in the above reaction is produced by α-elimination

(C) *meta*-hydroxy benzaldehyde is the major product

(D) *ortho*-hydroxy benzaldehyde is the major product

32. The correct statement (s) related to colloids is(are)

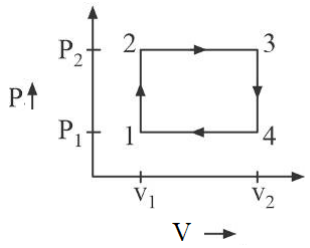
(A) Mists and fogs are the colloidal systems in which liquid is dispersed in a gaseous dispersion medium.

(B) Greater the valency of coagulating ion, the greater is © power of coagulation

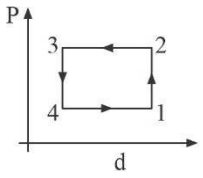
(C) Lyophilic sols have greater viscosity than pure water

(D) Micelles are formed by surfactant molecules above critical micelle concentration

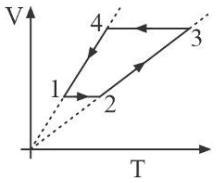
33. An ideal gas was subjected to change in state in cyclic manner $(P-V \text{ diagram})$. Which of following plot/s are correct with respect to the P–V diagram of the ideal gas sample. (Here P, V, T, d, S are pressure, volume, temperature density and entropy of the gas respectively).



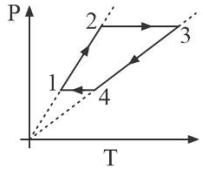
(A)



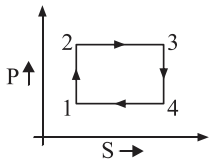
(B)



(C)



(D)



34. The correct statement(s) related to the metal extraction process is(are):

(A) In extraction of iron from haematite ore, the reduction reactions take place only in the lower temperature range in the blast furnace.

(B) Solidified copper obtained from silica lined convertor (Bessemer converter) has blistered appearance due to the evolution of SO_2

(C) In Serpek's process, silica is removed by heating the bauxite to 1800°C with coke in a current of N_2

(D) Extraction of zinc from sphalerite involves roasting followed by carbon reduction

35. Select correct statement regarding analysis of radicals.

(A) Presence of S^{2-} ions can be confirmed by using sodium nitroprusside.

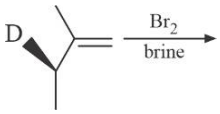
(B) Griss-llosway test is used for identification of NO_2^- ions.

(C) Red eosin dye test is used to confirm presence of Br^- ions.

(D) Brown ring test is not useful to identify presence of NO_3^- in solution.

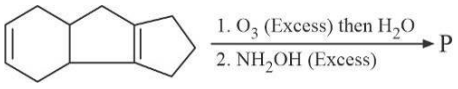
Section – 4 | Non-Negative Integer Type

This **Section** contains **3 Non-Negative Integer Type Questions.** The answer to each question is a **NON-NEGATIVE INTEGER.**

36. 

number of fractions containing only organic compounds obtained after fractional distillation of product mixture.

37. In the reaction given below, the total number of atoms having sp^2 hybridization in the major product P is _____.



38. If number of optically active isomers of $[Pt(gly)_2Cl_2]$ is X and number of optically active isomers of $[Co(en)_2Cl_2]$ is Y, then what is the value of (X) × (Y)?

SPACE FOR ROUGH WORK

SUBJECT III : MATHEMATICS 60 MARKS

Section – 1 | Single Correct Type

This Section contains **4 Single Correct Answer Type Questions**. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.

39. Let $P(1, 0)$ and $Q(0, 1)$ be two fixed points on the circle $x^2 + y^2 = 1$. Let R be a variable point on this circle. The locus of orthocentre of ΔPQR is:

(A) $x^2 + y^2 = 4$ **(B)** $x^2 + y^2 - 2x - 2y + 1 = 0$

(C) $x^2 + y^2 + 2x - 2y + 1 = 0$ **(D)** $x^2 + y^2 - x - y = 0$

40. Let $f(x)$ and $g(x)$ be two functions defined from $\mathbb{R} \to \mathbb{R}$ such that $f(x) = \cos x - \cos^3 x$ and $g(x) = x^2 - \frac{\pi^2}{4}$. If area of the region bounded by $y = f(x)$ and $y = g(x)$ is $\left(\frac{2}{3} + \frac{\pi^3}{a}\right)$. Then the number of ordered pairs (x, y) satisfying $xy = a$ (where $x, y \in Z$) is:

(A) 6 **(B)** 8 **(C)** 4 **(D)** 9

41. A die is weighted such that the probability of rolling the face numbered n is proportional to $n^2, (n = 1,2,3,4,5,6)$. The die is rolled twice, yielding the numbers a and b. If the probability that $a < b$ can be expressed in lowest rational as p/q then $(p + q)$ is:

(A) 177 **(B)** 213 **(C)** 316 **(D)** 124

42. If $z = \cos\frac{2\pi}{15} + i\sin\frac{2\pi}{15}$ (where $i = \sqrt{-1}$); then $\arg\left(\frac{1 + z + z^2 + z^3 + ... + z^7}{1 + z^8 + z^9 + ... + z^{14}}\right)$ is not equal to:

(A) $\frac{14\pi}{15}$ **(B)** $\pi - \frac{1}{2}\arg(z)$ **(C)** $2\pi + 8\arg(\bar{z})$ **(D)** $7\arg(\bar{z})$

Section – 2 | Numerical Value Type

This **Section** contains **3 Question stems**. There are **TWO (02)** questions corresponding to each question stem. For each question, enter the correct numerical value of the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/roundoff** the value to **TWO** decimal places.

Question Stem: For Question Nos. 43 to 44

Question Stem

A six digit number is formed randomly using digits (1, 2, 3) with repetitions. Then:

43. Probability that all digits are used at least once is p/q then $p + q$ where p, q are coprimes, is _____.

44. Probability that exactly two digits are used is a/b, a and b are coprimes then $b - a$ is _____.

Question Stem: For Question Nos. 45 to 46

Question Stem

$$\text{If } \begin{bmatrix} 4a^2 & 4a & 1 \\ 4b^2 & 4b & 1 \\ 4c^2 & 4c & 1 \end{bmatrix} \begin{bmatrix} f(-1) \\ f(1) \\ f(2) \end{bmatrix} = \begin{bmatrix} 3a^2+3a \\ 3b^2+3b \\ 3c^2+3c \end{bmatrix}$$

where $f(x) = ax^2 + bx + c$ (a, b, c, are distinct real numbers) whose maximum value occurs at a point V say (α, β). Let A be the point of intersection of $y = f(x)$ with negative x-axis, say (p, o) and point B is such that the chord AB subtends a right angle at V. Let B be (r, s). Let Δ be the area enclosed by $y = f(x)$ and the chord AB. Then :

45. Value of $(\alpha + \beta)$ is _____.

46. Value of (3Δ) is _____.

Question Stem: For Question Nos. 47 to 48

Question Stem

A straight line passing through $O(0, 0)$ cuts the lines $x = \alpha$, $y = \beta$ and $x + y = 8$ at A, B and C respectively such that $OA \cdot OB \cdot OC = 48\sqrt{2}$ and $f(\alpha, \beta) \le 0$,

where $f(x, y) = \left|\frac{y}{x} - \frac{3}{2}\right| + (3x - 2y)^6 + \sqrt{ex + 2y - 2e - 6}$.

47. $(OA + OB + OC)^2$ equals _____.

48. Slope of line OA is _____.

Section – 3 | Multiple Correct Type

This Section contains **6 Multiple Correct Answer Type Questions.** Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).

49. Which of the following is/are correct?

(A) If A is a $n \times n$ matrix such that $a_{ij} = (i^2 + j^2 - 5ij)\cdot(j - i)\ \forall\ i$ and j then trace $(A) = 0$

(B) If A is a $n \times n$ matrix such that $a_{ij} = (i^2 + j^2 - 5ij)\cdot(j - i)\ \forall\ i$ and j then trace $(A) \neq 0$

(C) If P is a 3×3 orthogonal matrix, α, β, γ are the angle made by a straight line with OX, OY, OZ and $A = \begin{bmatrix} \sin^2\alpha & \sin\alpha\cdot\sin\beta & \sin\alpha\cdot\sin\gamma \\ \sin\alpha\cdot\sin\beta & \sin^2\beta & \sin\beta\cdot\sin\gamma \\ \sin\alpha\cdot\sin\gamma & \sin\beta\cdot\sin\gamma & \sin^2\gamma \end{bmatrix}$ and $Q = P^T AP$, then

$PQ^6P^T = 32A$

(D) If matrix $A = [a_{ij}]_{3\times3}$ and matrix $B = [b_{ij}]_{3\times3}$ where $a_{ij} + a_{ji} = 0$ and $b_{ij} - b_{ji} = 0\ \forall\ i$ and j then A^7B^6 is a singular matrix

50. Which of the following is/are incorrect?

(A) Let $f : R \to R$, such that $f(x) = 2x + \left[\dfrac{x(x^2-1)}{4(x^4-x^2+1)} + \dfrac{1}{8}\right] + [x] + \sin x \cos x$, then (where $[\cdot]$ denotes the greatest integer function) f is bijective

(B) Let $f : R \to R$, such that $f(x) = \dfrac{x^3 + 2x^2}{\sin x + 2}$ then f is bijective

(C) Let $f : R \to [1, \infty)$ such that $f(x) = 2^{|x-1|^{\sqrt{2}}}$ then $f(x)$ is one-one into

(D) Let $f : R - \{0\} \to R$ such that $f(x) = |x|\ln|x|$ then $f(x)$ is one-one onto

51. A box contains 'm' black balls and 'n' red balls. All the balls except for one ball are drawn from it. Then the probability that the last ball remaining in the box is black is:

(A) $\dfrac{m}{m+n}$ when balls of same colour are identical

(B) $\dfrac{m}{m+n}$ when no two balls are identical

(C) 1/2 when no two balls are identical

(D) $\dfrac{m!}{(m+n)!}$ when no two balls are identical

52. If P is a non-null matrix of order 3×3 with real numbers as entries, such that $P^3 = O$, where O is a null matrix of order 3×3, then (where I is the identity matrix of order 3×3)

(A) det of $(4P^2 - 2P + I)$ is a non-zero number

(B) $I - 2P$ is an invertible matrix

(C) If matrix P has all integer entries, then $\det(I - 4P^2) = 0$

(D) If matrix P has all integer entries, then absolute value of $\det(I - 4P^2) = 1$

53. If $p = \sec^2(\tan^{-1}2) + \text{cosec}^2(\cot^{-1}3)$ and $q = \pi - \tan^{-1}\left(\frac{\sqrt{1+\tan^2 8}+1}{\tan 8}\right)$, then:

(A) $\cot^{-1}(p-3q) > \cot^{-1}(p-5q)$

(B) $\tan^{-1}(pq-20) < \tan^{-1}(p^2-2q^2)$

(C) $\sin^{-1}(\sin(pq+4))$ is a positive real number

(D) $\sin^{-1}(\sin pq)$ is a negative real number

54. Let $A(z_1), B(z_2), C(z_3)$ and $D(z_4)$ be the vertices of a trapezium in that order is an Argand plane such that $AB \parallel CD$. Let $|z_1 - z_2| = 4, |z_3 - z_4| = 10$ and the diagonals AC and BD intersect at P. It is given that $\text{Arg}\left(\frac{z_4 - z_2}{z_3 - z_1}\right) = \frac{\pi}{2}$ and $\text{Arg}\left(\frac{z_3 - z_2}{z_4 - z_1}\right) = \frac{\pi}{4}$. Then which of the following option(s) is/are correct?

(A) Area of the trapezium $ABCD$ is equal to $\frac{140}{3}$ sq. units

(B) Area of the trapezium $ABCD$ is equal to $\frac{70}{3}$ sq. units

(C) Value of $|CP - DP|$ is equal to $\frac{10}{\sqrt{21}}$

(D) $PC : AC = 2 : 3$

Section – 4 | Non-Negative Integer Type

This **Section** contains **3 Non-Negative Integer Type Questions.** The answer to each question is a **NON-NEGATIVE INTEGER.**

55. Let $P(x) = x^6 - x^5 - x^3 - x^2 - x$ and $Q(x) = x^4 - x^3 - x^2 - 1$. If α, β, γ and δ are the roots of $Q(x) = 0$, then the value of $P(\alpha) + P(\beta) + P(\gamma) + P(\delta)$ is equal to:

56. If r_1, r_2 and r_3 are ex-radii of triangle ABC, which are roots of $x^3 - 11x^2 + 36x - 36 = 0$ and I_1, I_2, I_3 are ex-centres opposite to vertex A, B, C respectively. The area of triangle $I_1I_2I_3$ is k, then the value of k/10 is:

57. Let $A(\vec{a})$ and $B(\vec{b})$ be two points on two skew lines $\vec{r} = \vec{a} + \lambda\vec{p}$ and $\vec{r} = \vec{b} + \mu\vec{q}$ and the shortest distance between the skew lines is 1 unit, where $\vec{p}$ and $\vec{q}$ are unit vectors adjacent sides of a parallelogram enclosing an area of 1/2 sq. units and λ, μ are scalar parameters. If angle between AB and the line of shortest distance is 60° and length of AB is k, then the value of k^4 is:

SPACE FOR ROUGH WORK